Глобальный фазовый портрет

$Y' = F(Y)$ $\quad$ $Y(t) = Y^*$ – положение равновесие, $F(Y^*) = 0$

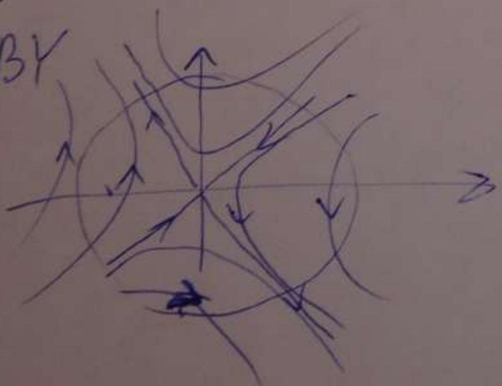
$$F(Y) = \underbrace{F(Y^*)}_{} + \underbrace{\frac{\partial F}{\partial Y}\Big|_{Y=Y^*}}_{} (Y - Y^*) + o(\|Y - Y^*\|)$$

$Y' = B(Y - Y^*) + o(\|Y - Y^*\|)$ В

$Y' = BY$

Утв. Пусть $\forall \lambda(B)$ $Re\lambda \neq 0$, тогда фазовый портрет сист $Y' = BY$
в некоторой окрестности Y^* качественно эквивалентен фазовому
портрету сист $Y' = F(Y)$, т.е. тип положение равновесие сохраняется
при линеаризации (качественно эквив – $\exists$ взаимно однозначное
отображение, переводящее траектории $Y' = BY$ в траект $Y' = F(Y)$,
сохраняющее направление движение)

$Y' = BY$



$Y' = F(Y)$

$$\begin{cases}\dot x = y\\ \dot y = x^2 - e^y\end{cases} \qquad \begin{cases} y = 0\\ x^2 - e^y = 0\end{cases} \qquad \begin{matrix} y = 0 \\ x^2 = 1\end{matrix} \Rightarrow \begin{matrix}(1,0)\\(-1,0)\end{matrix}$$

$$\frac{\partial F}{\partial Y} = \begin{pmatrix} 0 & 1 \\ 2x & -e^y \end{pmatrix}$$

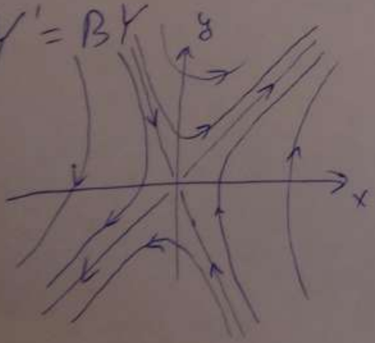
(1,0)

$$B = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} \quad \begin{matrix}\lambda_1 = -2 \\ \lambda_2 = 1\end{matrix} \text{ седло}$$

$$-2: \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \quad h_c = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

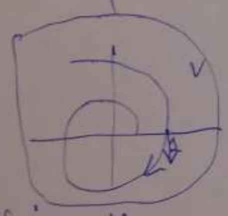
$$1: \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \quad h_c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

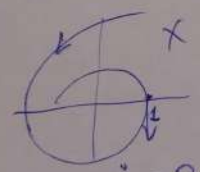
$Y' = BY$



(-1,0)

$$B = \begin{pmatrix} 0 & 1 \\ -2 & -1 \end{pmatrix} \quad \begin{matrix}\lambda_1 = -\frac{1}{2} + i\frac{\sqrt{7}}{2} \\ \lambda_2 = -\frac{1}{2} - i\frac{\sqrt{7}}{2}\end{matrix}$$





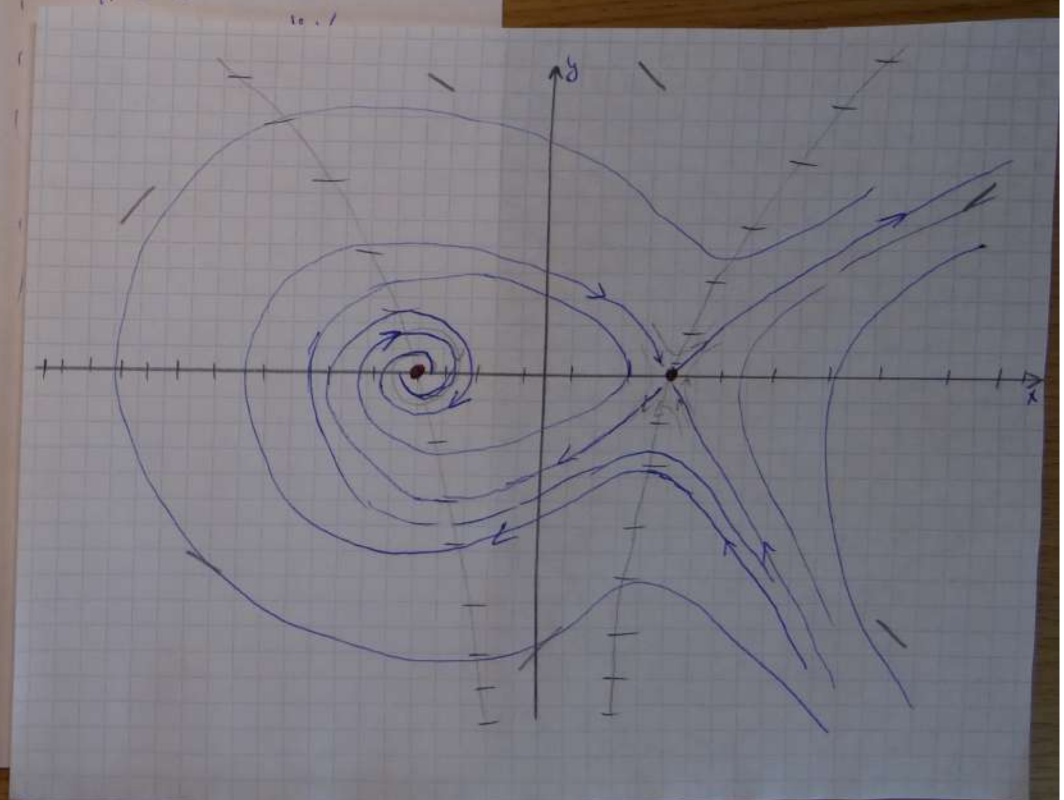
$$\begin{cases}\dot x = y\\ \dot y = -2x - y\end{cases}$$

$x = 1$, $y = 0$; $\dot x = 0$, $\dot y = -2$

$$\frac{dy}{dx} = \frac{x^2 - e^y}{y}$$

$y = 0$ – изокл ∞

$x^2 = e^y$ – изокл 0



$$\begin{cases}\dot{x} = x^2 - y^2 - 1 \\ \dot{y} = y\end{cases}$$

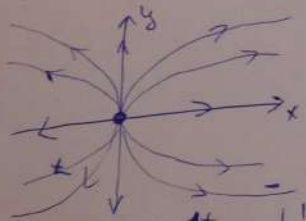
$$\begin{cases}x^2 - y^2 - 1 = 0 \\ y = 0\end{cases} \qquad \begin{cases}x = \pm 1 \\ y = 0\end{cases}$$

$$\frac{\partial F}{\partial Y} = \begin{pmatrix} 2x & -2y \\ 0 & 1 \end{pmatrix}$$

$(1,0)$ $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$\lambda_1 = 1 \quad \lambda_2 = 2$ - узел

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

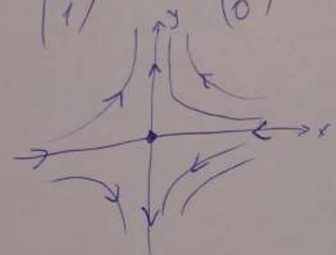


$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{1t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t}$$

$(-1,0)$ $B = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$

$\lambda_1 = 1 \quad \lambda_2 = -2$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

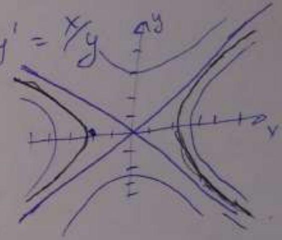


$y = 0$ - изокл 0

$x^2 - y^2 = 1$ - изокл ∞

$(-1,0)\ (1,0)$ - проходит ч/з точки

$2x - 2yy' = 0 \quad y' = x/y$

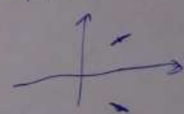


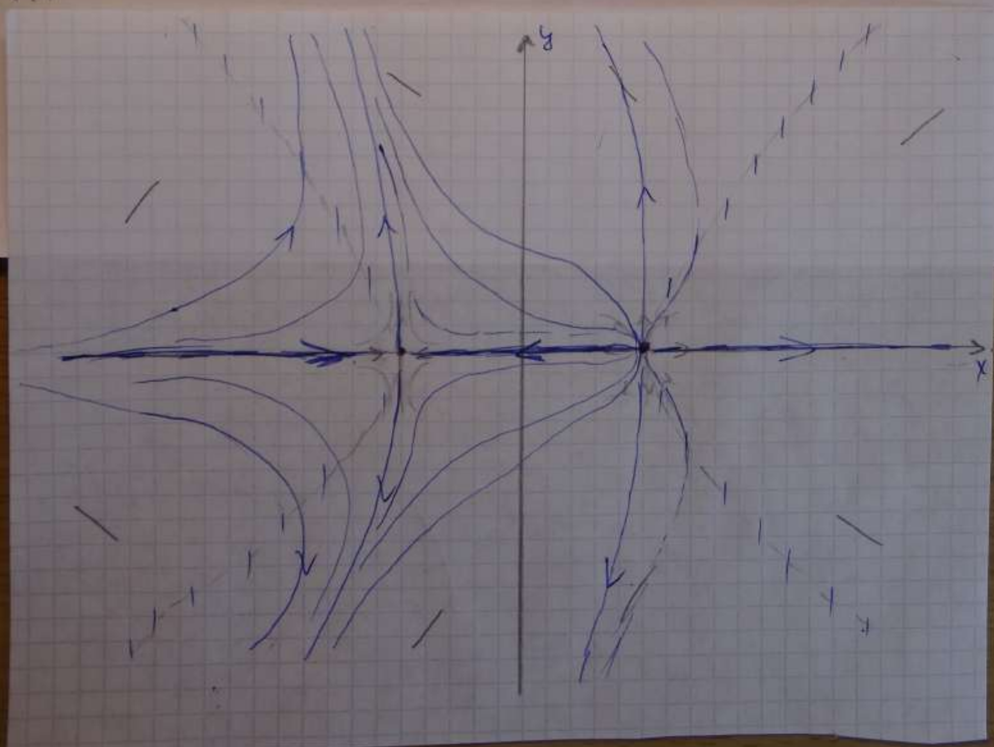
$y = x$

$y = -x$

$$\frac{dy}{dx} = \frac{y}{x^2 - y^2 - 1} = f(x,y)$$

$f(x,-y) = -f(x,y)$





1009 $\ddot{\varphi} - \sin\varphi = 0$

$$\begin{cases} x = \varphi \\ y = \dot{\varphi} \end{cases} \Rightarrow \begin{cases} \dot{x} = \dot{\varphi} = y \\ \dot{y} = \ddot{\varphi} = \sin\varphi = \sin x \end{cases}$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = \sin x \end{cases}$$

$y = 0$

$x = \pi k$

$$\frac{\partial F}{\partial Y} = \begin{pmatrix} 0 & 1 \\ \cos x & 0 \end{pmatrix}$$

$$\frac{dy}{dx} = \frac{\sin x}{y} = f(x, y)$$

$x = \pi k$ - изока 0

$y = 0$ - изока ∞

$f(-x, -y) = f(x, y)$

$f(x, -y) = -f(x, y)$

$f(-x, y) = -f(x, y)$

$k = 2n$

$(2\pi n; 0)$

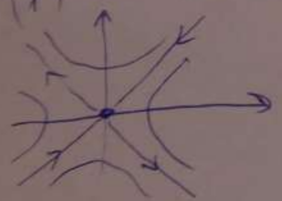
$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\lambda_1 + \lambda_2 = 0$

$\lambda_1 \cdot \lambda_2 = -1$

$\lambda_1 = 1;\ \lambda_2 = -1$ - седло

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$



$k = 2n + 1$

$(2\pi n + \pi, 0)$

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$\lambda_1 + \lambda_2 = 0$

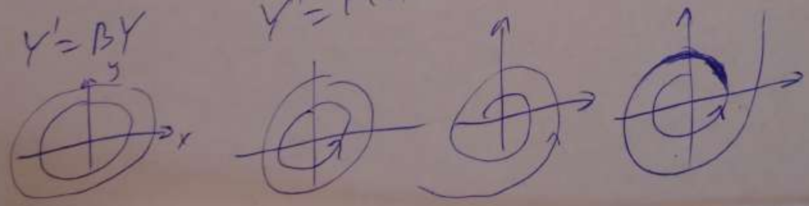
$\lambda_1 \cdot \lambda_2 = 1$

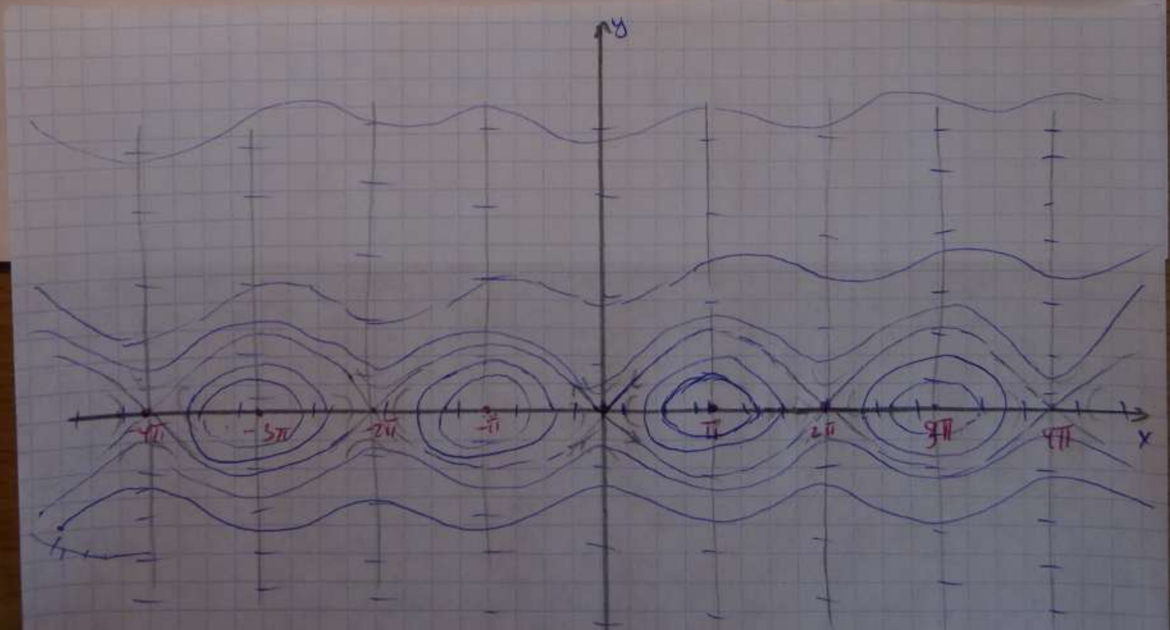
$\lambda = \pm i$.

$Y' = BY$

$\mathrm{Re}\,\lambda = 0$

$Y' = F(Y)$





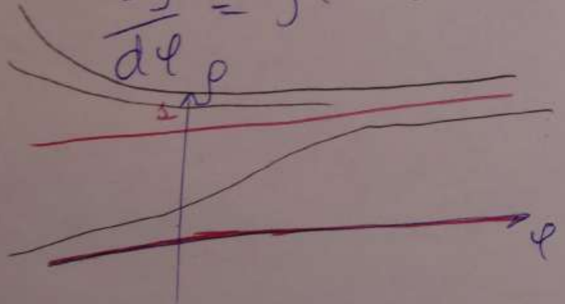
$$\begin{cases} x' = -y + x(1-x^2-y^2) \\ y' = x + y(1-x^2-y^2) \end{cases}$$

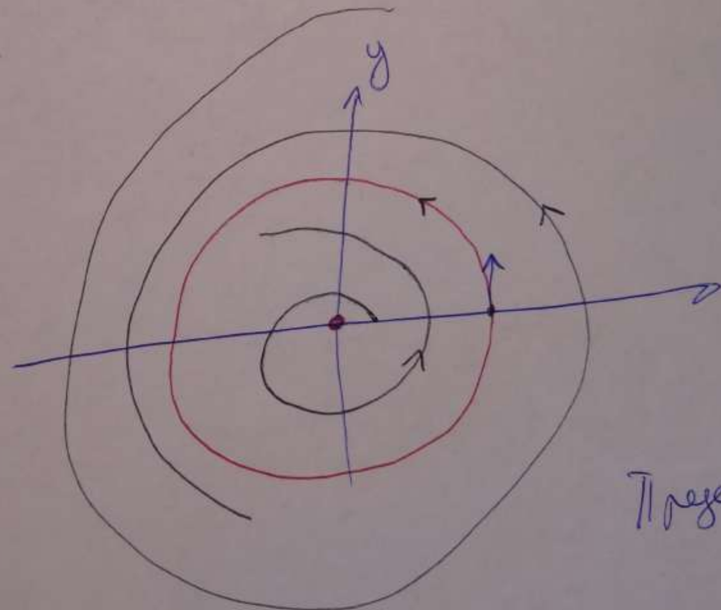
$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases}$ - замена $\rho(t), \varphi(t)$

$$\begin{cases} \rho'\cos\varphi - \rho\sin\varphi\cdot\varphi' = -\rho\sin\varphi + \rho\cos\varphi(1-\rho^2\cos^2\varphi-\rho^2\sin^2\varphi) \\ \rho'\sin\varphi + \rho\cos\varphi\cdot\varphi' = \rho\cos\varphi + \rho\sin\varphi(1-\rho^2\cos^2\varphi-\rho^2\sin^2\varphi) \end{cases} \quad \left|\begin{matrix}\cos\varphi \\ + \\ \sin\varphi\end{matrix}\right. \quad \left|\begin{matrix}\sin\varphi \\ - \\ \cos\varphi\end{matrix}\right.$$

$$\begin{cases} \rho' = \rho(1-\rho^2) \\ -\rho\varphi' = -\rho \end{cases} \qquad \begin{cases} \rho' = \rho(1-\rho^2) \\ \varphi' = 1 \end{cases}$$

$$\frac{d\rho}{d\varphi} = \rho(1-\rho^2)$$





$\begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases}$

$x = 1$
$y = 0$

$\begin{cases} \dot{x} = 0 \\ \dot{y} = 1 \end{cases}$

Предельный цикл устойчив

$\begin{cases} x' = x - y \\ y' = x + y \end{cases}$ $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $\lambda_1 + \lambda_2 = 2$

$\lambda_1\lambda_2 = 2$

$\lambda_1 = 1 + i$

$\lambda_2 = 1 - i$