1162, 1147, 1154, 1153, 1155, 1167, 1169, 1171, 1189, 1192.

(1) $\frac{\partial u}{\partial t} + f_1(t, y_1, \dots y_n)\frac{\partial u}{\partial y_1} + \dots + f_n(t, y_1, \dots, y_n)\frac{\partial u}{\partial y_n} = 0$ – УЧИ линейное, однородное 1-го порядка

$u = u(t, y_1, \dots, y_n)$

(2) $\begin{cases} \frac{dy_1}{dt} = f_1(t, Y) \\ \dots \\ \frac{dy_n}{dt} = f_n(t, Y) \end{cases}$

<u>Опр.</u> Ф-ие $\varphi(t, y_1, \dots y_n) \not\equiv const$ – <u>первый интеграл сист (2)</u>, если $\forall$ реш $Y(t)$ сист (2) $\varphi(t, y_1(t), \dots y_n(t)) \equiv const$.

<u>Замеч</u> Глад. ф-ие $\varphi(t, Y)$ – $IS(2) \Rightarrow \frac{d\varphi(t, Y(t))}{dt} = 0$, где $Y(t)$ – произв реш (2)

<u>Th</u>, $\varphi(t, Y)$ – $IS(2) \Leftrightarrow \varphi(t, Y)$ – реш УЧИ (1).

<u>Th.</u> Сист (2) имеет n функц. незав. IS, $\forall$ др $IS(2)$ φ:

$\varphi = G(\varphi_1, \dots \varphi_n)$, G – произв глад ф-ие.

<u>Опр.</u> $\varphi_1(t, Y), \dots \varphi_k(t, Y)$ – $IS(2)$, они <u>функционально независимы</u>, если

$\operatorname{rank} \begin{pmatrix} \frac{\partial \varphi_1}{\partial y_1} & \dots & \frac{\partial \varphi_1}{\partial y_n} \\ \dots & & \dots \\ \frac{\partial \varphi_k}{\partial y_1} & \dots & \frac{\partial \varphi_k}{\partial y_n} \end{pmatrix} = k$

| т.е. $\neg\exists\, W$: $\varphi_1 = W(\varphi_2, \dots \varphi_k)$

$\boxed{1162}$ $\begin{cases} \dot{x} = xy \\ \dot{y} = x^2+y^2 \end{cases}$ $\varphi_1(t,x,y) = x\ln y - x^2 y$ — не IS

$\varphi_2(t,x,y) = y^2/x^2 - 2\ln x$ — IS сист.

$$\frac{d\varphi_1}{dt} = \dot{x}\ln y + x\frac{1}{y}\dot{y} - 2x\dot{x}y - x^2\dot{y} = xy\cdot\ln y + \frac{x}{y}(x^2+y^2) - 2x^2y^2 - x^2(x^2+y^2) \neq 0$$

$$\frac{d\varphi_2}{dt} = \frac{2y\cdot\dot{y}x^2 - y^2\dot{x}2x}{x^4} - 2\frac{\dot{x}}{x} = \frac{2y(x^2+y^2)x^2 - y^2xy2x}{x^4} - 2y = 2y-2y = 0$$

Св-во равных дробей:

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \dots = \frac{a_k}{b_k} = p \Rightarrow \forall \alpha_1,\dots\alpha_k \quad \frac{\alpha_1 a_1 + \alpha_2 a_2 + \dots + a_k\alpha_k}{\alpha_1 b_1 + \alpha_2 b_2 + \dots + \alpha_k b_k} = p = \frac{a_i}{b_i}$$

$\boxed{1147}$ $$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}.$$

$$\begin{cases} \dfrac{dz}{dx} = \dfrac{z}{y} \\ \dfrac{dy}{dx} = \dfrac{x}{y}. \end{cases}$$

$z = z(x)$

$y = y(x)$

$$\frac{dx}{y} = \frac{dy}{x}$$

$x\,dx = y\,dy$

$x^2 - y^2 = C_1$

$\varphi_1(x,y,z) = x^2 - y^2$

$$\frac{1\,dx + 1\,dy}{1\cdot y + 1x} = \frac{dz}{z}$$

$$\frac{d(x+y)}{x+y} = \frac{dz}{z}$$

$x + y = C_2\cdot z$

$\varphi_2(x,y,z) = \dfrac{x+y}{z}$

$y^2 = x^2 - C_1$

$y = \pm\sqrt{x^2 - C_1}$

$z = \dfrac{x+y}{C_2}$

$z = \dfrac{x \pm \sqrt{x^2 - C_1}}{C_2}$

реш: $\begin{cases} y = \pm\sqrt{x^2 - C_1} \\ z = \dfrac{x \pm \sqrt{x^2 - C_1}}{C_2} \end{cases}$

$\boxed{1154}$ $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{dz}{xy+z}$

$\dfrac{dx}{x} = \dfrac{dy}{y} \Rightarrow x = Cy \Rightarrow \varphi_1(x,y,z) = x/y$;

$\dfrac{dy}{y} = \dfrac{dz}{xy+z}$; $\dfrac{dy}{y} = \dfrac{dz}{cy^2+z}$; $\dfrac{dz}{dy} = cy + z/y$; $z = z(y)$

$z_{oo} = C_2 y$

$z_{гоот} = C_2(y)y$

$C_2' y = Cy$

$C_2'(y) = C$

$C_2(y) = Cy + C_2^0$

$z(y) = Cy^2 + C_2^0 y$

$C_2^0 = \dfrac{z - Cy^2}{y} = \dfrac{z-xy}{y}$

$\varphi_2(x,y,z) = \dfrac{z - xy}{y}$

$\dfrac{y\,dx + x\,dy - dz}{yx + xy - xy - z} = \dfrac{dy}{y}$; $\dfrac{d(xy-z)}{xy-z} = \dfrac{dy}{y}$ $\quad xy - z = Cy$

$\varphi_3(x,y,z) = x - \dfrac{z}{y}$;

Реш: $\begin{cases} x = Cy \\ xy - z = Cy \end{cases}$

$\boxed{1153}$

$$\frac{dx}{z^2-y^2} = \frac{dy}{z} = \frac{dz}{-y}$$

$$\frac{dy}{z} = \frac{dz}{-y}$$

$$y\,dy + z\,dz = 0$$

$$y^2+z^2 = C_1$$

$$\varphi_1(x,y,z) = y^2+z^2$$

$$\frac{dx}{z^2-y^2} = \frac{z\,dy + y\,dz}{z^2-y^2}$$

$$dx = z\,dy + y\,dz$$

$$C_2 = zy - x$$

$$\varphi_2(x,y,z) = -x+zy$$

$\boxed{1155}$

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy\sqrt{z^2+1}}$$

$$\frac{dx}{xz} = \frac{dy}{yz}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

$$x = Cy$$

$$\varphi_1(x,y,z) = x/y$$

$$\frac{y\,dx + x\,dy}{2xyz} = \frac{dz}{xy\sqrt{z^2+1}}$$

$$d(xy) = \frac{2z\,dz}{\sqrt{z^2+1}}$$

$$xy = \frac{\sqrt{z^2+1}}{2} + C$$

$$\varphi_2(x,y,z) = 2xy - \sqrt{z^2+1}$$

1167 $y z_x - x z_y = 0 \quad z = z(x,y) \qquad y \neq 0: \; z_x - \frac{x}{y} z_y = 0$

$\frac{dy}{dx} = -\frac{x}{y} \qquad x^2+y^2 = C; \quad \Phi(x,y) = x^2+y^2; \quad z = G(x^2+y^2)$, – G - $\forall$ гл. ф-я

1169 $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0 \quad u = u(x,y,z)$

$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}; \qquad \frac{dx}{x} = \frac{dy}{y} \qquad x = Cy \qquad \varphi_1(x,y,z) = x/y$

$\frac{dy}{y} = \frac{dz}{z} \qquad y = Cz \qquad \varphi_2(x,y,z) = y/z$

$\forall$ I S $\varphi = G(x/y, y/z) \Rightarrow u = G(x/y; y/z)$, G - произв. гл.

$G(z_1,z_2) = z_1 \cdot z_2 + e^{z_1 \cdot \cos z_2} \Rightarrow$

$u = \frac{x}{z} + e^{\frac{x}{y}\cdot \cos \frac{y}{z}}$ – тоже реш., частное реш.

1171 $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x - y; \quad z = z(x,y); \quad z = z_{oo} + z_{част}$

$z_{oo}: \; y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0 \qquad z_{oo} = G(x^2-y^2); \quad z_{част} = y - x;$

$\frac{dx}{y} = \frac{dy}{x} \qquad z = G(x^2-y^2) + y - x$, G - $\forall$ гл.

$x^2 - y^2 = C$

$\Phi(x,y) = x^2 - y^2$

$\boxed{1189}$ $\begin{cases} x z_x - y z_y = 0 \\ z|_{y=1} = 2x \end{cases}$

$x z_x - y z_y = 0$

$\frac{dx}{x} = \frac{dy}{-y}$

$xy = C$

$\varphi(x,y) = xy$

$z = G(xy)$

$2x = G(x) \Rightarrow G(xy) = 2xy$

$\Rightarrow z = 2xy$ – реш.

$\boxed{1192}$ $\begin{cases} u_x + u_y + 2u_z = 0 \\ u|_{z=0} = x^2 + y^2 \end{cases}$

$u_x + u_y + 2u_z = 0.$

$dx = dy = \frac{dz}{2}$

$x - y = C_1 \qquad 2y - z = C_2$

$\varphi_1(x,y,z) = x - y \; ; \quad \varphi_2(x,y,z) = 2y - z$

$u = G(x-y; 2y-z)$

$x^2 + y^2 = G(\underbrace{x-y}_{s}; \underbrace{2y}_{t})$

$\frac{t}{2} = y$

$s + \frac{t}{2} = x$

$x^2 + y^2 = \left(\frac{t}{2}\right)^2 + \left(s + \frac{t}{2}\right)^2$

$G(s,t) = \frac{t^2 + (2s+t)^2}{4}$

$u = \frac{(2y-z)^2 + (2(x-y) + 2y - z)^2}{4}$

$u = \frac{(2y - z)^2 + (2x - z)^2}{4}$ – реш. ЗК.